d-dimensional oscillating scalar field lumps and the dimensionality of space

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Extremely long-lived, time-dependent, spatially-bound scalar field configurations are shown to exist in d spatial dimensions for a wide class of polynomial interactions parameterized as $V(\phi) = \sum_{n=1}^{h} \frac{g_n}{n!}\phi^n$. Assuming spherical symmetry and if $V'' < 0$ for a range of values of $\phi(t,r)$, such configurations exist if: i) spatial dimensionality is below an upper-critical dimension d_c; ii) their radii are above a certain value $R_{\rm min}$. Both d_c and $R_{\rm min}$ are uniquely determined by $V(\phi)$. For example, symmetric double-well potentials only sustain such configurations if $d \leq 6$ and $R^2 \geq d[3(2^{3/2}/3)^d - 2]^{-1/2}$. Asymmetries may modify the value of d_c. All main analytical results are confirmed numerically. Such objects may offer novel ways to probe the dimensionality of space.

INTRODUCTION

It is well known that many physical systems may be efficiently modeled in a reduced number of spatial dimensions. In particular, certain static solutions of nonlinear classical field equations exhibiting solitonic behavior have been used to describe a wide variety of phenomena, ranging from hydrodynamics and condensed matter physics [1] to relativistic field theories [2]. At the opposite extreme, the possibility that the four fundamental interactions may be unified in theories with extra spatial dimensions has triggered research on the existence of static nonperturbative solutions of nonlinear field theories in more than three spatial dimensions [3] [4]. These extra dimensions may be compact and much smaller than the usual three dimensions of space, as in Kaluza-Klein (KK) theories [5], or they may be infinitely large, as in the recent Randall-Sundrum (RS) proposal, where gravity (and possibly other fields) can leak into the extra dimension transverse to the 3-dimensional brane where matter and gauge fields propagate [6]. There have been many variants of the RS proposal [7], including some with more than one brane [8] or with more than a single large extra dimension [9]. In either the KK or the RS scenarios, there is plenty of motivation to study d-dimensional nonperturbative field configurations with a large number of quanta N, even if their direct production in particle colliders is probably exponentially suppressed as $\sim \exp[-N]$. [There has been much interest recently in the possibility that extra dimensions could produce signatures observable in collider experiments [10] [11] [12] [13] [14] [15]. Although I will not examine if the objects of the present work could be produced in future collider experiments, the possibility should be kept in mind, especially with more realistic models involving couplings between several fields.]

In this work, I argue that long-lived *time-dependent* d-dimensional scalar field lumps – *oscillons* – can exist in a wide class of models, much wider than their static (solitonic) counterparts. Furthermore, I show that they exist only below a certain critical number of spatial dimensions, which is uniquely determined by the field's self-interactions. If the fundamental gravity scale is M, the associated length scale of the extra dimensions is $R_{\rm KK} \simeq (M_{\rm Pl}/M)^{2/(d-3)}M^{-1}$. [$d-3 \geq 1$ is the number of extra dimensions.] Thus, if $M \sim 1$ TeV, $R_{\rm KK} \sim 10^{32/(d-3)}10^{-17}$ cm. For $d \geq 5$, this scenario is still acceptable by current tests of Newton's gravitational law [7]. Even though oscillons ultimately decay, their lifetimes are long enough to produce significant effects: their demise occurs in very short time-scales and hence would appear, in the scenario with large but compact extra dimensions, as a sudden burst of particles from a small region. If these particles are quanta of the scalar field, their masses would satisfy, assuming maximally-symmetric internal dimensions, $p_\mu p^\mu + n^2/R^2_{\rm KK} = V''(\phi_v)^{1/2}$, where $\mu = 0, 1, 2, 3$, and $V''(\phi_v)^{1/2}$ is the tree-level mass of vacuum excitations satisfying the d-dimensional Klein-Gordon equation. The key point here is that since the mass and size of d-dimensional oscillons are uniquely determined by the number of spatial dimensions and their interaction potential, they can serve as probes to the dimensionality of space. A d-dimensional oscillon hypothetically appearing at the TeV energy scale will have a typical size of order d TeV$^{-1} \sim 10^{-17}$ cm, always much smaller than $R_{\rm KK}$.

So far, most work in either a reduced or increased number of spatial dimensions has focused on *static* solutions involving real scalar fields or scalar fields coupled to other fields. A recent example is the work by Bazeia et al., where d-dimensional spherically-symmetric topological defects were found for models with potential $U(x^2;\phi) = f(x^2)V(\phi)$ [16]. The particular choice of potential is needed to evade Derrick's theorem, which forbids the existence of non-trivial static solutions for real scalar fields in more than one spatial dimension [17]. When time-dependence is introduced, it is often as a general phase of a complex scalar field, $\phi(x,t) = \varphi(x)\exp[-i\omega t]$, such that the equations still allow for localized solutions with static spatial profiles. Nontopological solitons [18] and Q-balls [19] are well-known examples of such configurations. There are exceptions, though. Breathers in one dimension [20], and oscillons in two [21] [22] [23] and three [24] [25] [26] [27] [28] are spatially-bound, time-dependent scalar field configura-

tions which are remarkably long-lived. They are found in many physical systems and models, including vibrating grains, Josephson junctions, nonlinear Schrödinger equations, Ginzburg-Landau models, and certain relativistic ϕ^4 models, to name a few examples. As will be seen, they also exist in higher-dimensional models for a wide class of polynomial interactions.

SCALAR FIELD DYNAMICS IN D-DIMENSIONS

The line element for flat $d+1$-dimensional spacetime is $ds^2 = \eta_{MN}dx^M dx^N$, where $M, N = 0, 1, 2, ..., d$ and $\eta_{MN} = \mathrm{diag}(+, -, -...., -)$. I'm only concerned here with objects which may exist in the full d dimensions. Thus, their typical size $R_{\rm min}$ will have to satisfy $R_{\rm min} \ll R_{\rm KK}$, where $R_{\rm KK}$ is the linear size of the extra dimensions.

Since any deformation away from spherical symmetry leads to more energetic configurations [23] [19], I will consider only spherically-symmetric configurations, $\phi(t, r)$. As such, the d-dimensional spatial volume element can be written as $d^d x = c_d r^{(d-1)} dr$, where $c_d = 2\pi^{d/2}/\Gamma(d/2)$ is the surface area of a d-dimensional sphere of unit radius. The Lagrangian can be written as

$$L = c_d \int r^{(d-1)} dr \left(\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\left(\frac{\partial \phi}{\partial r}\right)^2 - V(\phi) \right) , \quad (1)$$

where a dot means time derivative.

Previous results in $d = 2$ and $d = 3$ have shown that oscillons are well-approximated by configurations with a general Gaussian profile [25] [21] [22]. In fact, oscillons can be viewed as localized configurations modulated by nonlinear oscillations on the field's amplitude. I will thus treat the amplitude as a function of time, keeping the radius R constant. The fact that the analytical results to be obtained are verified numerically to great accuracy confirms that this approximation is adequate for this work's purpose. Oscillons are thus modeled as

$$\phi(t, r) = [\phi_c(t) - \phi_v] \exp[-r^2/R^2] + \phi_v \ , \quad (2)$$

where $\phi_c(t)$ is the core value of the field $[\phi(t, r = 0)]$, and ϕ_v is its asymptotic value at spatial infinity, determined by $V(\phi)$. Thus, one condition on $V(\phi)$ is that $\partial^2 V/\partial\phi^2|_{\phi_v} > 0$. Note also that the equation of motion for $\phi(t, r)$ imposes that $\phi'(r = 0) = 0$, a condition satisfied by the Gaussian ansatz above.

Since many applications involve polynomial potentials, we will write $V(\phi)$ as

$$V(\phi) = \sum_{j=1}^{h} \frac{g_j}{j!}\phi^j - V(\phi_v) \ , \quad (3)$$

where the g_j's are constants. The vacuum energy $V(\phi_v)$ is subtracted from the potential to avoid spurious divergences upon spatial integration.

Substituting the ansatz of eq. (2) into eq. (1), one may perform the spatial integrations to obtain,

$$L[A, R, \dot{A}] = \left(\frac{\pi}{2}\right)^{d/2} R^d \left[\frac{\dot{A}^2}{2} - \frac{d}{2R^2}A^2 - \sum_{n=2}^{h} \left(\frac{2}{n}\right)^{d/2} \frac{1}{n!} V^n(\phi_v) A^n \right] , \quad (4)$$

where $V^n(\phi_v) \equiv \frac{\partial^n V(\phi_v)}{\partial \phi^n}$, and I introduced the amplitude $A(t) \equiv \phi_c(t) - \phi_v$. Note that the sum in the last term starts at $n = 2$. This is due to the fact that, by definition, $\frac{\partial V(\phi_v)}{\partial \phi} = 0$.

UPPER CRITICAL DIMENSION FOR OSCILLONS

From the Lagrangian in eq. (4) one obtains the equation of motion for $A(t)$:

$$\ddot{A} = -\frac{d}{R^2}A - \sum_{n=2}^{h} \left(\frac{2}{n}\right)^{d/2} \frac{1}{(n-1)!} V^n(\phi_v) A^{n-1} \ . \quad (5)$$

If $V(\phi) = 0$, the amplitude undergoes harmonic oscillations with constant frequency $\omega^2 = \frac{d}{R^2}$. This behavior is due to the surface term that resists any displacement from equilibrium, $A = 0$. Note that since the Lagrangian was integrated over all space, the model cannot describe the fact that the configuration decays by radiating its energy to spatial infinity. If needed, one could include a phenomenological term $\gamma\dot{A}$ in order to mimic this effect (such as $\gamma \sim t^{-3/2}$ in $d = 3$ [25]), although this is not relevant for the present work.

To examine the stability of the motion, I expand the amplitude as $A(t) = A_0(t) + \delta A(t)$. Linearizing eq. (5),

$$\delta\ddot{A} = -\left[\frac{d}{R^2} + \sum_{n=2}^{h} \left(\frac{2}{n}\right)^{d/2} \frac{1}{(n-2)!} V^n(\phi_v) A_0^{n-2} \right] \delta A \equiv -\omega^2(R, A_0)\delta A \ , \quad (6)$$

where I introduced the effective frequency ω^2 in the last line. Instabilities occur if $\omega^2 < 0$. Long-lived oscillons are only possible if the oscillations above the vacuum with amplitude $A(t)$ probe regions of $V'' < 0$ for a sustained period of time [24] [25]. This requires $\omega^2 < 0$ for oscillons to exist.

I proceed by deriving several results from the expression for ω^2. First, it is useful to write it fully as,

$$\omega^2(R, A_0) = \frac{d}{R^2} + V''(\phi_v) + \left(\frac{2}{3}\right)^{d/2} V'''(\phi_v) A_0 + \left(\frac{1}{2}\right)^{d/2} V^{IV}(\phi_v) A_0^2 + \left(\frac{2}{5}\right)^{d/2} \frac{1}{3!} V^V(\phi_v) A_0^3 + \cdot \quad (7)$$

Note that since A_0 is a function of time, eq. (6) is in the form of a Mathieu equation. Although the time dependence is crucial in the study of oscillon dynamics (c.f. ref. [22]), it will not be relevant here.

Quadratic potentials

If $V(\phi)$ is quadratic, only the first two terms on the RHS of eq. (7) contribute to ω^2. For $V''(\phi_v) > 0$, $\omega^2 > 0$ and no instability occurs [25]. The field will simply undergo damped oscillations about $A = 0$. If $V''(\phi_v) < 0$, instabilities are possible only for $d/R^2 < |V''(\phi_v)|$ or

$$R \geq \left(\frac{d}{|V''(\phi_v)|}\right)^{1/2} . \tag{8}$$

This is the well-known spinodal instability bound [29], where the critical wavelength λ_c is related to the initial size of the configuration, R, by $R = (\sqrt{d}/2\pi)\lambda_c$. When the condition for instability is satisfied, the amplitude $A(t)$ will grow without bound as the field rolls down the potential. This behavior will depend on the damping term, that is, on the rate at which the lump radiates energy to spatial infinity. If the condition eq. (8) is *not* satisfied, the amplitude will describe damped oscillations about the origin. Note that this can only happen in a field theory, since the gradient term is needed to allow for $\omega^2 > 0$ even if $V''(\phi_v) < 0$. In effect, the gradient term stabilizes what would have been an unstable configuration. This mechanism is favored as d increases, as one would expect.

Gaussian-shaped bubbles with quadratic potentials are thus short-lived, not a surprising result [25]: oscillons owe their longevity to nonlinearities in the potential. Furthermore, as it is proven next, a necessary condition for their existence is that the potential satisfies $V''(A) < 0$ for at least a range of amplitudes. This is also true for solitons, which do not exist for potentials with positive concavity, e.g., $V(\phi) = \phi^2 + \phi^4$. This necessary condition, however, is not sufficient to guarantee the existence of oscillons.

Cubic potentials

If $V(\phi)$ is cubic, the first thing to notice is that since parity is broken, $V(\phi)$ will always have an inflection point at $\phi_{\rm inf} = -g_2/g_3$. The choice of vacuum will depend on the sign of g_2: for $g_2 > 0$, $\phi_v = 0$; for $g_2 < 0$, $\phi_v = -2g_2/g_3$. In either case, the condition $\omega^2 < 0$ will be satisfied whenever A_0 has opposite sign to g_3 *and* for values of $R \geq R_{\rm min}$ as summarized in Table 1.

	$g_3 > 0$	$g_3 < 0$
$g_2 > 0$	$\frac{d}{g_3\left[\left(\frac{2}{3}\right)^{d/2}\lvert A_0\rvert+\phi_{\rm inf}\right]}$	$\frac{d}{\lvert g_3\rvert\left[\left(\frac{2}{3}\right)^{d/2}A_0-\phi_{\rm inf}\right]}$
$g_2 < 0$	$\frac{d}{g_3\left[\left(\frac{2}{3}\right)^{d/2}\lvert A_0\rvert-\phi_{\rm inf}\right]}$	$\frac{d}{\lvert g_3\rvert\left[\left(\frac{2}{3}\right)^{d/2}A_0+\phi_{\rm inf}\right]}$

Table 1: Values of $R^2_{\rm min}$ for different couplings of the cubic potential model.

Since $R^2 > 0$, the amplitudes must satisfy, for any of the cases in Table 1,

$$|A_0| \geq \left(\frac{3}{2}\right)^{d/2} |\phi_{\rm inf}| , \tag{9}$$

showing that only fluctuations probing deep into the spinodal region of the potential will be able to sustain long-lived oscillons. It is also clear that the higher the dimensionality the larger the amplitudes need to be. However, for cubic potentials, as long as the conditions above are satisfied, long-lived oscillating lumps can exist in any number of dimensions. This result does not hold for arbitrary polynomial potentials, as we see next.

Quartic potentials

For quartic potentials, the condition for the existence of oscillating lumps becomes,

$$\begin{aligned}\omega^2(R, A_0) \;&\leq\; \frac{d}{R^2} + V''(\phi_v) + \left(\frac{2}{3}\right)^{d/2} V'''(\phi_v)A_0 \\ &+\; \frac{1}{2}\left(\frac{1}{2}\right)^{d/2} V^{IV}(\phi_v)A_0^2 \; .\end{aligned} \tag{10}$$

Results are sensitive to the sign of $V^{IV}(\phi_v) = g_4$. Let me first examine the case for $g_4 > 0$: ω^2 is a parabola with positive concavity. Thus, if $\omega^2 < 0$ at its minimum, the condition is satisfied for a range of amplitudes. The minimum is at $\bar{A}_0 = -(4/3)^{d/2}V'''/V^{IV}$, and

$$\omega^2(R, \bar{A}_0) = \frac{d}{R^2} + V'' - \left(\frac{2}{3}\right)^d 2^{(d-2)/2}\frac{(V''')^2}{V^{IV}} \; . \tag{11}$$

For $\omega^2 < 0$,

$$R^2 \geq \frac{d}{\left[\frac{1}{2}\left(\frac{2^{3/2}}{3}\right)^d \frac{(V''')^2}{V^{IV}} - V''\right]} \; . \tag{12}$$

So, as in the case for cubic potentials, oscillating lumps can only exist for radii above a critical size. This has been observed numerically for double-well potentials in $d = 2$ [21] and $d = 3$ [24]. Notice also that since the denominator must be positive definite, this condition imposes

both a restriction on the potential *and* an upper critical dimension for oscillons:

$$d \leq \mathrm{Int}\left[\frac{\ln 2\frac{V''V^{IV}}{(V''')^2}}{\ln\left(\frac{2^{3/2}}{3}\right)}\right] , \tag{13}$$

where Int means the integral part of. Equality defines the upper critical dimension for the existence of oscillons, d_c. Since $\frac{2^{3/2}}{3} < 1$, the potential must satisfy

$$2\frac{V''V^{IV}}{(V''')^2} < 1 , \tag{14}$$

for the bifurcation instability on the radius to occur and, thus, for oscillating lumps to exist.

If $g_4 < 0$, $\phi_v = 0$ for stability. In this case, $g_2 > 0$ (the sign of g_3 is irrelevant) and there will always be an amplitude $|A_0|$ large enough so that $\omega^2 < 0$,

$$A_0 = 2^{d/2}\left(\frac{2}{3}\right)^{d/2}\frac{g_3}{|g_4|}\times \tag{15}$$
$$\left[1 \pm \left(1 + 3^d 2^{-3d/2} 2\left(g_2 + \frac{d}{R^2}\right)\frac{|g_4|}{g_3^2}\right)\right] .$$

Comparing this amplitude with the value for the inflection point, $\phi_{\mathrm{inf}} = g_3/|g_4|\left[1 \pm \left(1 + 2g_2|g_4|/g_3^2\right)\right]$, it's easy to see that $|A_0| > |\phi_{\mathrm{inf}}|$, that is, oscillons only exist if the amplitudes go beyond the inflection point.

APPLICATION: SYMMETRIC AND ASYMMETRIC DOUBLE-WELL POTENTIALS IN D-DIMENSIONS

For a symmetric double-well (SDW) potential with $g_4 > 0$, $V(\phi) = \frac{\lambda}{4}\left(\phi^2 - \phi_v^2\right)^2$, the coefficients of the general expression eq. (3) are; $h = 4$, $g_1 = g_3 = 0$, $g_2 = -\lambda\phi_v^2$, $g_4 = 6\lambda$, and thus the various derivatives at ϕ_v are $V'(\phi_v) = 0$, $V''(\phi_v) = 2\lambda\phi_v^2$, $V'''(\phi_v) = 6\lambda\phi_v$, $V^{IV}(\phi_v) = 6\lambda$. The necessary condition eq. (14) is satisfied, $2V''V^{IV}/(V''')^2 = 2/3 < 1$. From eq. (12), oscillons will exist if the radius is larger than [the radius can be made dimensionless with the scaling $R = R'/\sqrt{\lambda}\phi_v$ for any d]

$$R^2 \geq \frac{d}{\left[3\left(\frac{2^{3/2}}{3}\right)^d - 2\right]} . \tag{16}$$

For $d = 2$, $R_{\mathrm{min}} = \sqrt{3}$ [21]. For $d = 3$, $R_{\mathrm{min}} \simeq 2.42$ [25]. The expression predicts that, e.g., for $d = 6$, $R_{\mathrm{min}} \simeq 7.5$. Note also that the SDW has an upper critical dimension of $d_c = 6$: from eq. (13), $d \leq \mathrm{Int}\left[\ln(2/3)/\ln(2^{3/2}/3)\right] = 6$.

I have confirmed these results numerically, using a leap-frog method fourth-order accurate in space. The lattice spacing was $\delta r = 0.01$ and the time step $\delta t = 0.001$.

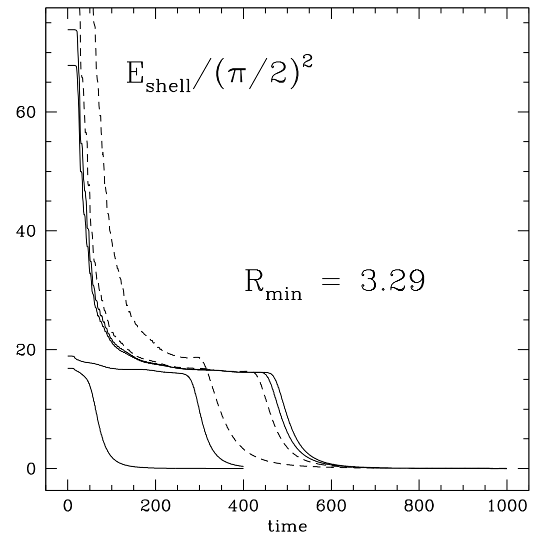

FIG. 1: Time evolution of the energy within a shell of radius $R_{\mathrm{shell}} = 10R$ in $d = 4$. From left to right: continuous lines are for $R_{\mathrm{eff}} = 3.17, 3.29, 4.81, 4.93$; dashed lines are for $R_{\mathrm{eff}} = 5.87, 5.17$. The plateaus denote oscillons.

Energy was conserved to better than one part in 10^5. The program solves the d-dimensional Klein-Gordon equation in spherical coordinates with initial condition set to be a Gaussian bubble with radius R and $\phi_c = 1$ and $\phi_v = -1$. [One can vary the initial profile and parameters at will; if the conditions for the appearance of oscillons are satisfied, the field will evolve into an oscillon configuration, since it is an attractor in field-configuration space [23] [25]. Furthermore, oscillons have been shown to emerge even from thermal initial states [22].] The program reproduced results from refs. [21] and [24] in $d = 2$ and $d = 3$, respectively. In Fig. 1, I show the energy within a shell of radius $R_{\mathrm{shell}} = 10R$ as a function of time for $d = 4$ ($R_{\mathrm{min}} \simeq 3.29$). The approximately flat plateaus denote oscillons in 5-dimensional spacetime. As in $d = 2$ and $d = 3$, there is a range of values of R that lead to oscillons: larger values produce configurations that decay without settling into an oscillon.

The effective radius of the configuration, the one checked against the predictions of eq. 16, is computed as the normalized first moment of the energy distribution,

$$R_{\mathrm{eff}} = \frac{\int r^d dr\left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\phi'2 + V(\phi)\right]}{\int r^{(d-1)} dr\left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\phi'2 + V(\phi)\right]} . \tag{17}$$

In Fig. 2, I show similar results for $d = 6$, again confirming the prediction of eq. (16). [The very narrow low-energy plateaus seen in Fig. 2 seem to be a peculiarity of the $d = 6$ case. Given that this feature is irrelevant for the main arguments of this work, I will not investigate it

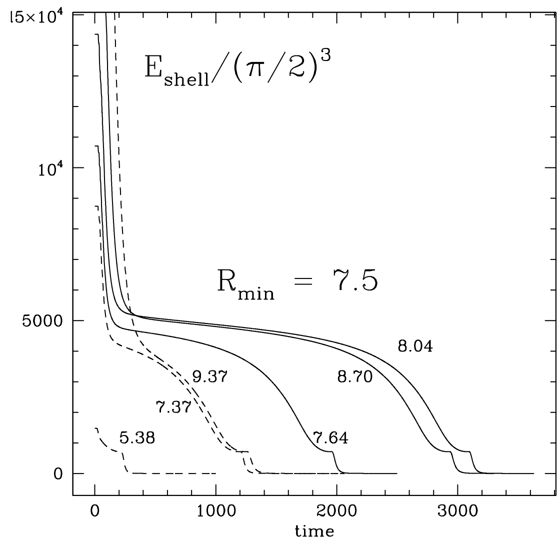

FIG. 2: Time evolution of the energy within a shell of radius $R_{\rm shell} = 10R$ in $d = 6$. The labels specify the initial value of $R_{\rm eff}$. Continuous lines are for oscillons; dashed lines are for failed configurations.

further.] For $d = 7$ and larger, I was unable to find any oscillons, confirming that $d_c = 6$ for SDW potentials.

For an "upside-down" SDW, $g_4 < 0$. Writing the potential as $V(\phi) = \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\phi^4$, and scaling the field as $X = \phi/\phi_v$, with $\phi_v = \sqrt{6/\lambda}m$, oscillons will exist as long as the amplitude of oscillations satisfies $X^2 > 2^{d/2}\left(1 + d/R^2\right)X^2_{\rm inf}$. $X_{\rm inf}$ is the inflection point.

For asymmetric double-well potentials the situation changes. From eq. (13) it is easy to see that the upper critical dimension d_c can vary as a function of the asymmetry: the larger the absolute value of the asymmetry, the higher d_c. As an illustration, consider the potential

$$V(\phi) = \frac{1}{2}\phi^2 - \frac{\alpha}{3}\phi^3 + \frac{1}{4}\phi^4 \ , \tag{18}$$

where $\alpha > 0$ and the variables are all dimensionless. Oscillons with lifetimes of order $10^4{\rm m}^{-1}$ have been found for this model with $d = 3$ [25]. The necessary condition eq. (14) gives $\alpha^2 > 3$, which is the same that guarantees an inflection point for $V(\phi)$. The condition for upper critical dimension reads, $d \leq {\rm Int}[\ln(3/\alpha^2)/\ln(2^{3/2}/3)]$. For example, $\alpha^2 = 9/2$, equivalent to a SDW, gives $d_c = 6$ as it should. $\alpha^2 = 5$ gives $d_c = 8$. Thus, asymmetries may relax (but not eliminate) the bound on the upper critical dimension for oscillons.

For SDW potentials, one can introduce dimensionless variables $r' \equiv \sqrt{\lambda}\phi_v r$, $t' \equiv \sqrt{\lambda}\phi_v t$, $X = \phi/\phi_v$ such that the energy scales as $E[\phi] = \lambda^{(2-d)/2}|\phi_v|^{4-d}E[X]$. From the numerical results obtained, a rough (within a factor of 2) estimate of their energies in d dimensions is $E[X] \sim \left(\frac{\pi}{2}\right)^{d/2}\frac{1}{2}d^{d-1}$. Of course, it's always possible to obtain accurate results numerically, as shown in Figs. 1 and 2 (in units of c_d) for $d = 4$ and $d = 6$, respectively: an oscillon in 5 dimensional spacetime would have an energy of $E/c_5 \simeq 20\lambda^{-1}$. The characteristic length-scale of d-dimensional oscillons is determined by eq. (16). Again, a rough estimate gives, $R_{\rm min} \sim \frac{d}{\sqrt{\lambda}\phi_v}$. It is straightforward to extend these arguments to arbitrary potentials.

The results of this work have established that if a potential can support oscillons, they will have a well-defined set of properties which are dimensionally-dependent: their energies [the approximately flat plateaus of Figs. 1 and 2] and their average radii. Also, the minimum radius for the initial configurations that lead to oscillons is determined by the dimensionality of space, as seen for the SDW potential in eq. (16). Thus, one can envision that if such configurations were to be observed, and if the interactions were known, their energies and sizes would uniquely determine the dimensionality of space. In the example of Fig. 2 above ($d = 6$), with a vacuum scale of 1 TeV, a typical oscillon will have a radius $R_{\rm eff} \sim 10^{-17}$ cm, while $R_{\rm KK} \sim 5 \times 10^{-7}$. In this case, a flat space approximation such as the one used here would be quite acceptable, and the observed masses would receive only slight corrections from the extra dimensions. A next step would be to examine if these configurations exist for models with several interacting fields, including those carrying Abelian and non-Abelian quantum numbers. It may be possible to find long-lived d-dimensional Q-balls in φ^4 models. If $d = 3$, this includes the Standard Model and its supersymmetric extensions. Also, an estimate of the oscillon lifetime and its dependence on spatial dimensionality is still lacking. (Note how $d = 6$ oscillons live four to five time longer than those in $d = 4$.)

Relaxing the constraint of having static, spatially-localized solutions to the equations of motion opens many avenues for further investigation: as was shown in this work, long-lived *time-dependent* localized configurations are supported by a wide class of models. They may not only be observed in $d = 3$ but also offer a new window into the extra dimensions, in case they exist.

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